Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics
# Standard level
# Paper 2

Wednesday 4 November 2020 (morning)

Candidate session number

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1 hour 30 minutes

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

12 pages

8820 – 7302

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

**1.** [Maximum mark: 6]

Consider the function $f(x) = x^2 + x + \frac{50}{x},\ x \neq 0$.

(a) Find $f(1)$. [2]

(b) Solve $f(x) = 0$. [2]

The graph of $f$ has a local minimum at point $\mathrm{A}$.

(c) Find the coordinates of $\mathrm{A}$. [2]

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**2.** [Maximum mark: 6]

Lucy sells hot chocolate drinks at her snack bar and has noticed that she sells more hot chocolates on cooler days. On six different days, she records the maximum daily temperature, $T$, measured in degrees centigrade, and the number of hot chocolates sold, $H$. The results are shown in the following table.

| **Maximum temperature ($T$)** | 14 | 8 | 4 | 18 | 13 | 11 |
|---|---|---|---|---|---|---|
| **Number of hot chocolates ($H$)** | 79 | 143 | 191 | 58 | 84 | 105 |

The relationship between $H$ and $T$ can be modelled by the regression line with equation $H = aT + b$.

(a) (i) Find the value of $a$ and of $b$.

(ii) Write down the correlation coefficient. [4]

(b) Using the regression equation, estimate the number of hot chocolates that Lucy will sell on a day when the maximum temperature is $12^{\circ}\text{C}$. [2]

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**3.** [Maximum mark: 6]

A discrete random variable $X$ has the following probability distribution.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | $q$ | $4p^2$ | $p$ | $0.7 - 4p^2$ |

(a) Find an expression for $q$ in terms of $p$. [2]

(b) (i) Find the value of $p$ which gives the largest value of $\mathrm{E}(X)$.

(ii) Hence, find the largest value of $\mathrm{E}(X)$. [4]

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**4.** [Maximum mark: 7]

Let $f(x) = 4 - x^3$ and $g(x) = \ln x$, for $x > 0$.

(a) Find $(f \circ g)(x)$. [2]

(b) (i) Solve the equation $(f \circ g)(x) = x$.

(ii) Hence or otherwise, given that $g(2a) = f^{-1}(2a)$, find the value of $a$. [5]

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**5.** [Maximum mark: 6]

Consider the expansion of $\left(3x^2 - \frac{k}{x}\right)^9$, where $k > 0$.

The coefficient of the term in $x^6$ is $6048$. Find the value of $k$.

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**6.** [Maximum mark: 8]

An infinite geometric series has first term $u_1 = a$ and second term $u_2 = \frac{1}{4}a^2 - 3a$,

where $a > 0$.

(a) Find the common ratio in terms of $a$. [2]

(b) Find the values of $a$ for which the sum to infinity of the series exists. [3]

(c) Find the value of $a$ when $S_\infty = 76$. [3]

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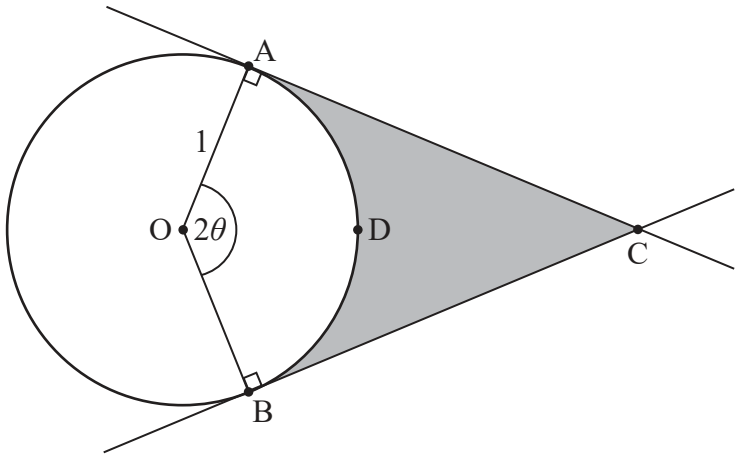
**7.** [Maximum mark: 7]

The following diagram shows a circle with centre O and radius $1\,\text{cm}$. Points A and B lie on the circumference of the circle and $\text{A}\hat{\text{O}}\text{B} = 2\theta$, where $0 < \theta < \frac{\pi}{2}$.

The tangents to the circle at A and B intersect at point C.

**diagram not to scale**




(a) Show that $\text{AC} = \tan\theta$. [1]

(b) Find the value of $\theta$ when the area of the shaded region is equal to the area of sector OADB. [6]

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Do **not** write solutions on this page.

## Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

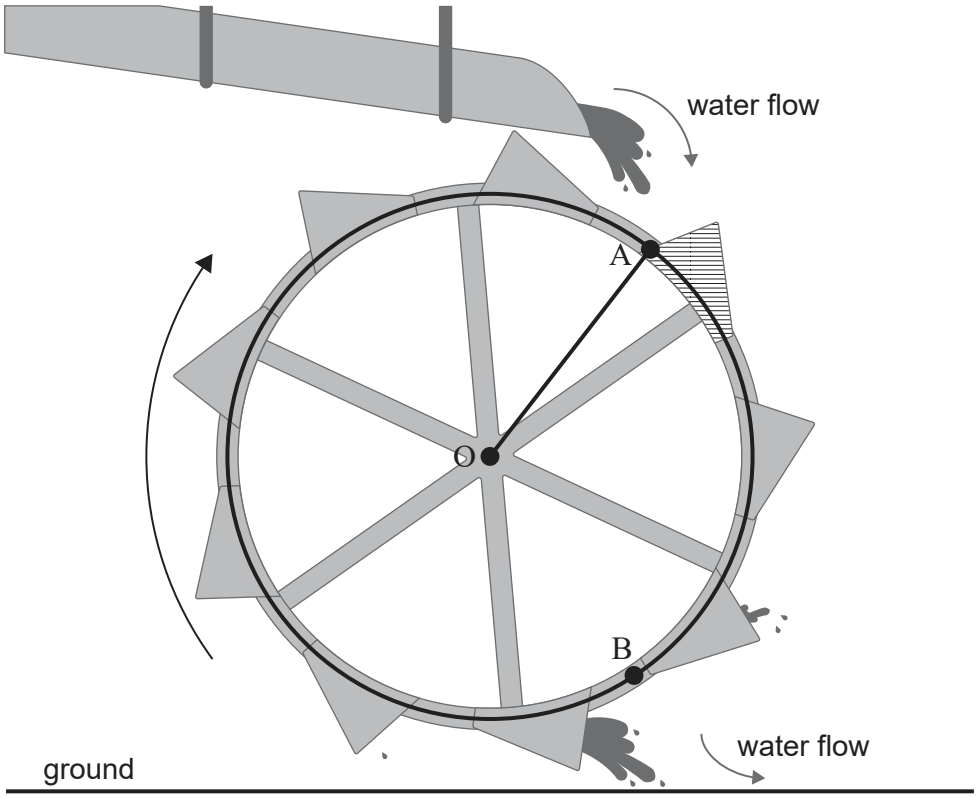
**8.** [Maximum mark: 14]

The following diagram shows a water wheel with centre O and radius 10 metres. Water flows into buckets, turning the wheel clockwise at a constant speed.

The height, $h$ metres, of the top of a bucket above the ground $t$ seconds after it passes through point A is modelled by the function

$$h(t) = 13 + 8\cos\left(\frac{\pi}{18}t\right) - 6\sin\left(\frac{\pi}{18}t\right), \text{ for } t \geq 0.$$

**diagram not to scale**




(a) (i) Find the height of point A above the ground.

(ii) Calculate the number of seconds it takes for the water wheel to complete one rotation.

(iii) Hence find the number of rotations the water wheel makes in one hour. [6]

**(This question continues on the following page)**

Do **not** write solutions on this page.

**(Question 8 continued)**

A bucket moves around to point $\mathrm{B}$ which is at a height of $4.06$ metres above the ground. It takes $k$ seconds for the top of this bucket to go from point $\mathrm{A}$ to point $\mathrm{B}$.

(b) Find $k$. [3]

The chord $[\mathrm{AB}]$ is $17.0$ metres, correct to three significant figures.

(c) Find $\mathrm{A\hat{O}B}$. [3]

(d) Determine the rate of change of $h$ when the top of the bucket is at $\mathrm{B}$. [2]

Do **not** write solutions on this page.

**9.** [Maximum mark: 14]

Fiona walks from her house to a bus stop where she gets a bus to school. Her time, $W$ minutes, to walk to the bus stop is normally distributed with $W \sim \mathrm{N}(12, 3^2)$.

Fiona always leaves her house at $07{:}15$. The first bus that she can get departs at $07{:}30$.

(a) Find the probability that it will take Fiona between $15$ minutes and $30$ minutes to walk to the bus stop. [2]

The length of time, $B$ minutes, of the bus journey to Fiona's school is normally distributed with $B \sim \mathrm{N}(50, \sigma^2)$. The probability that the bus journey takes less than $60$ minutes is $0.941$.

(b) Find $\sigma$. [3]

(c) Find the probability that the bus journey takes less than $45$ minutes. [2]

If Fiona misses the first bus, there is a second bus which departs at $07{:}45$. She must arrive at school by $08{:}30$ to be on time. Fiona will not arrive on time if she misses both buses. The variables $W$ and $B$ are independent.

(d) Find the probability that Fiona will arrive on time. [5]

This year, Fiona will go to school on $183$ days.

(e) Calculate the number of days Fiona is expected to arrive on time. [2]

**10.** [Maximum mark: 16]

Consider a function $f(x)$, for $x \geq 0$. The derivative of $f$ is given by $f'(x) = \dfrac{6x}{x^2 + 4}$.

(a) Show that $f''(x) = \dfrac{24 - 6x^2}{(x^2 + 4)^2}$. [4]

The graph of $f$ is concave-down when $x > n$.

(b) Find the least value of $n$. [2]

(c) Find $\int \dfrac{6x}{x^2 + 4} \mathrm{d}x$. [3]

Let $R$ be the region enclosed by the graph of $f$, the $x$-axis and the lines $x = 1$ and $x = 3$. The area of $R$ is $19.6$, correct to three significant figures.

(d) Find $f(x)$. [7]